GRS MA782- Hypothesis testing

Department of Mathematics and Statistics, Boston University, Spring 2016

Homework 4

1. Let X be a $N(\mu, 1)$ random variable and consider an estimator of μ of the form $\alpha + \beta X$. Let $R(\mu; \alpha, \beta)$ be the risk of $\alpha + \beta X$ with respect to the squared error loss function.

   1. Show that $\alpha + \beta X$ is inadmissible if $\beta > 1$ or if $\beta < 0$.
   2. Show that $\alpha + X$ is inadmissible for any $\alpha \neq 0$.
   3. Is α admissible?
   4. Show that $\alpha + \beta X$ is admissible for $0 < b < 1$ by slowing that it is Bayes with respect to a suitable prior $\pi(\mu)$.

2. Assume that $Z_1, \cdots, Z_n$ comes from a continuous population that is symmetric about a common median θ. Assume that the CDF of any of the Z_i's is the same and is denoted by F. If we have $n = 15$, $F = N(\mu, 16)$, then what is the approximate power of the level $\alpha = 0.076$ test for $H_0 : \theta = 0$ versus the alternative $H_1 : \theta > 0$ when treatment effect is $\theta = 1.25$?

3. Consider a level $\alpha = 0.25$ test of $H_0 : \theta = 0$ versus the alternative $H_1 : \theta > 0$ based on the Fisher sign test $B = \sum_{i=1}^{n} \chi_{\{Z_i > 0\}}$. If our data $Z_1, \cdots, Z_n$ is a random sample from a single continuous distribution $F(\cdot)$, then how many observations n, will we need to collect in order to have approximate power at least 0.75 against an alternative for which $F(0) = 0.20$? [Hint: read the paper by Noether, 1987 "Sample size determination for some common nonparametric tests", Journal of American Statistical Association, **82**, pp. 645-647.]

4. Consider the Wilcoxon two sample rank sum statistic W. Order the combined sample of $N = m + n$, X and Y values from least to greatest. Let S_i be the rank of the Y_i in this joint ordering. Set $W = \sum_{i=1}^{n} S_i$.

   1. Show that the minimum possible value of W is $\frac{n(n+1)}{2}$ whereas the maximum possible value of W is $\frac{n(2m+n+1)}{2}$.
   2. Suppose you reject $H_0 : \Delta = EY - EX = 0$, if $W = \frac{n(2m+n+1)}{2}$ or if $W = \frac{n(n+1)}{2}$ and accept H_0 otherwise. What is the type I error α for this test?