MA573 Qualitative Theory of Ordinary Differential Equations

Ryan Goh

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Course Details: Time: Monday-Wednesday-Friday, 10:10am-11:00am;

Location: CAS 314

Instructor: Ryan Goh

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Website: `http://math.bu.edu/people/rgoh/ma573-fs18/ma573-fs18.html`

Office: MCS 237B

Office hours: See course webpage

Course Textbook S. Strogatz, *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*, Studies in Nonlinearity. Westview Press; 2nd Edition. 2014.

You should be able to access an online version of the book via the BU library website.

Course Description: An introductory course in nonlinear dynamical systems. We shall focus on how to qualitatively analyze solutions of ordinary differential equations, even when solutions cannot be explicitly written down. I will assume that you have a solid background in Calculus, up to and including Multivariable Calculus (BU's MA 123-124-225 or 230), linear algebra (BU's MA 242 or 442) and that you have had a "Sophomore level" differential equations class (at the level of BU's MA 226 or MA 231). Necessary topics and tools from these fields will be recalled, briefly reviewed, and sometimes extended as needed. If you want any extra references to help review these courses please do not hesitate to contact me.

We hope to cover

- Introduction to nonlinear dynamics and qualitative analysis (Existence and uniqueness, phase portraits, stability, etc...)
- Basic bifurcations
- Basic oscillatory behavior
- Linear system theory in both two and higher dimensions
- Phase plane/portrait analysis (linear verus nonlinear stability, index theory, invariant manifolds)
- Asymptotic dynamics and limit-cycles (Conservered quantities, gradient flows, symmetry, lyapunov functionals, Poincare-Bendixson theory, Hopf bifurcation).
- An introduction to chaos theory
- (time permitting) a current overview of the field and where it's being applied

One of the beautiful things about the theory of dynamical systems is how it has been applied in many scientific disciplines. We aim to highlight as many applications as possible throughout this course. If you have a specific field which you are interested in, please contact me and I will try to work related examples into my lectures! Some possible areas of application include:

- Pattern formation in nature
- Chemical oscillations
- Singular pertubations
- Quantum systems
- Celestial mechanics
- Climate tipping points
- Fluid convection rolls
- Soliton waves
- Coupled oscillators and biological systems
- Models for infections disease
- Chaos and Cryptography

You will also have to solve differential equations on the computer during this course. As a BU student you have free access to both Mathematica and Matlab which would be sufficient for these tasks. There are also many free online tools (such as `http://math.rice.edu/~dfield/dfpp.html`) which one can use. One is also welcome to use Python or C++ paired with matplotlib (or other plotting software) to write code. If you haven't already used these softwares during your studies, need guidance on finding, installing, or using any of them, please feel free to ask me!

Homework Guidelines Doing exercises and problems is the best way to learn mathematics (and it is the best way to study for the midterms and exams). As such we will have weekly homework assignments. These will be due at the beginning of class one week afterwards. You are welcome (and encouraged!) to work together, but please make sure to write up solutions by yourself and *in your own words*. You must understand how to solve the problems yourself. If it is suspected that you just copied down your answers from another source of any kind, you may be asked to come to my office and explain your solution(s), without any books or notes.

In your solution sets, explain your work clearly, concisely, and using complete sentences. Homework should be legible and turned in on stapled pages. If using notebook paper please remove any frills. If you are worried about the legibility of your handwriting, please contact me about using the mathematical typesetting software LaTeX. I will pick 4 problems at random to grade from each problem set. See the grading section below for the how problems will be scored.

Midterm There will be one in-class midterm during the semester. It will be held on **Wednesday, October 24, during the class period**. It will consist of problems similar to the homeworks and in-class examples.

The content covered in this exam will be clearly defined one week in advance.

Final The final will be take-home, with problems being emailed out on Monday, December 17th. Problems will be similar to the those of the midterm, homeworks, and in-class examples. Logistics of the final will be discussed a few weeks prior. **No collaboration, or use of the internet is allowed. You are allowed to consult your class notes, textbook, and any course handouts, as well as use any computer codes for plotting phase portraits you may have written up over the span of the semester.**

Grades Your course grade will be based on your homework scores (50%), your midterm (20%) and your final (30%).

The majority of the homework problems (and some on the midterm and final) will be graded using the following 6-point system adapted from Prof. Chad Topaz, of Williams College:

- Mathematical content (4 points)
  - 4 points - The solution 100% correct.
  - 3 points - The solution shows a good understanding of the problem but there are some minor mistakes and/or typos present.
  - 2 points - The solution is incorrect but has some good ideas in it.
  - 1 points - A serious effort has been made on the problem
  - 0 points - Only a trivial, or no effort has been made
- Communication (2 points) These will only be assigned if you get 2 points or higher on the mathematical content portion.
  - 2 points - The solution is clearly written, well-organized, and justified with sufficient detail.
  - 1 point - The solution is readable, but is missing details or is hard to follow in places.
  - 0 points - The solution is unreadable, or has is missing many details.

No late homework will be accepted for any reason. The two lowest scores will be dropped at the end of the semester.

Other References Here are some other useful references. All of these have been put on reserve at the BU Science & Engineering Library. More references and links to specific papers relevant to lectures will be posted on the course webpage.

- (*) L. Perko, Lawrence. *Differential equations and dynamical systems.* Vol. 7. Springer Science & Business Media, 2013. - In between an undergraduate and graduate level text, has a nice review of the theory of finite dimensional homogeneous linear equations.
- (*) Hirsch, Morris W., Stephen Smale, and Robert L. Devaney. Differential equations, dynamical systems, and an introduction to chaos. Academic press, 2012. – A good undergraduate level reference with many nice examples.
- (*) Hale, Jack K., and Kocak, H.. Dynamics and bifurcations. Vol. 3. Springer Science & Business Media, 2012.
- (*) Arnold, V. I. Ordinary differential equations (translated by RA Silverman). MIT Press, Cambridge, Massachusetts 2 (1973): 196-213.

Make-up tests/Exams I do not provide make-up exams unless you have a legitimate excuse, such as serious illness, family emergency, or religious observance. In such a circumstance you must provide written excuse and, if possible, contact me far in advance. A make-up test may involve solving problems in front of me in my office on the board.

Extra Help If you feel you are falling behind in the course please do not hesitate to contact me! It is always easier to address misunderstandings sooner than later. In addition to working with me, I can also put you in touch with TAs in the Math Tutoring room who are knowledgeable in dynamical systems theory.

Academic Conduct Don't cheat. Don't plagarize. Please refer to the BU code of academic conduct if you are unclear on what constitutes appropriate behavior.