MA573 - Fall 2022 Homework 2 - Due September 16th

Hirsh-Smale-Devaney Problems

Chapter 1: 2(c),2(d), 3(b),(c), 5,11,12,

Additional Problem(s):

Problem 1: We wish to test the Euler method on the simple initial value problem x' = x, x(0) = 1.

- (i) Find the solution analytically, and calculate the exact value of x(1).
- (ii) Write up Euler's method for this equation in the coding language of your choice. With step size $\Delta t = 1$, estimate x(1) numerically; call it $\tilde{x}_0(1)$. Repeat using the step-sizes $\Delta t = 10^{-n}$ to obtain new approximations $\tilde{x}_n(1)$, of x(1) for n = 1, 2, 3, 4, 5
- (iii) Plot the error, $E = |\tilde{x}_n(1) x(1)|$, between approximate and actual solution as function of the step-size Δt . Now plot log E vrs. log t and explain the results.
- (iv) Repeat parts (ii) (iii) for the Improved Euler's method and the Runge-Kutta method, comparing all three methods.
- (v) (optional, but will be useful later on) Also try using one of MATLAB's built in solver packages such as "ode45" (which is a fancier version of the Runge-Kutta method) or "ode15s."

Provide all of your codes at the end of your write-up.