MA573 - Fall 2022 Homework 4 - Due September 30th

Hirsh-Smale-Devaney Problems

Chapter 2: 9

Chapter 3: 2(i), (iii),(iv), 5, 7, 16 (hint: first draw the phase portraits for both cases: $x' = +y$ and $x' = -y$ and think about when the functions $g(y) = \pm y$ coincide with the function $f(y) = |y|$.)

Non-textbook Problem(s):

Problem 1: Consider again the system from Problem 3.16 above:

$$\begin{aligned} x' &= |y| \\ y' &= -x. \end{aligned} \tag{0.1}$$

Use a numerical approach (such as Matlab's built in ODE solver "ode45" (or "ode15s")) to numerically solve the equation, plotting several solution trajectories $\{(x(t), y(t), \mid t \in \mathbb{R}\}$ to compare with your answer to 16 above. Some useful initial conditions to try would be points of the form $(c, -1)$, with c ranging from -3 to 3.

Problem 2:*(Projective coordinates) Consider the linear system*

$$\begin{aligned} x' &= x, \\ y' &= -y. \end{aligned}$$

For any line $\ell \subset \mathbb{R}^2$ passing through the origin, there exists a vector $V = (v_1, v_2)^T \in \mathbb{R}^2$ such that

$$\ell = \text{Span}\{V\} = \{c(v_1, v_2)^T \mid c \in \mathbb{R}\}.$$

(i) Give a qualitative argument for how the initial conditions on ℓ evolve as time increases. In other words what does the set of points $\ell_t = \{(\phi(t, X_0)) : X_0 \in \ell\}$ look like as time evolves.

(ii) Let's study this by deriving a differential equation for the slope of the line. Let $z(t) = y(t)/x(t)$, where y and x solve the linear system. Derive a differential equation for $z(t)$.

(iii) What are the asymptotics of solutions of the z-equation you found. Interpret this in terms of the evolution of lines ℓ under the flow of the original linear system.