MA573 - Fall 2022 Homework 9 - Due November 11th

Hirsh-Smale-Devaney Problems

Chapter 8: 2 (hint: see examples in Sec. 8.3 on pg. 173, as well as in 8.1 pg).

Non-textbook Problem(s):

Problem 1: Consider the linear system

$$
\begin{aligned}
x' &= x - x^3, \\
y' &= -y - x^2.
\end{aligned}
\tag{0.1}
$$

(i) Find all equilibria and classify each associated linear system. If possible, use the Linearization Theorem (pg. 168 of text) to also classify and make a rough sketch the nonlinear phase portrait in a local neighborhood of each equilibrium.

(ii) The origin should be a saddle. Describe the stable and unstable subspaces of it's linearization.

(iii) Using the method we discussed in class, find the 4th-order Taylor expansion of the function $h^u(x)$ which describes the local unstable manifold $W^u(0) = \{(x, y) \,|\, x = h^u(y)\}$. Use this to more accurately sketch the local phase portrait near the origin.

(iv) Do the same for the stable manifold $W^s(0)$. Note here the graph function should have the form $x = h^s(y)$.

Problem 2: Now consider the slightly altered system

$$
\begin{aligned}
x' &= x - x^3, \\
y' &= -y - x^3.
\end{aligned}
\tag{0.2}
$$

The linearization about the origin should not change. Does the unstable manifold $W^s(0)$ change?

Problem 3: Now investigate the above two systems numerically and compare the results with your calculations of the invariant manifolds. For example, how do solutions behave if you put initial conditions $(x_0, 1)$ for x_0 just off of the predicted stable manifold?